DOCUMENT RESUME

ED 301 438 SE 050 157

TITLE	Introduction to Calculus 305--Optional Half Credit. Interim Guide.
INSTITUTION	Manitoba Dept. of Education, Winnipeg.
REPORT NO	ISBN-0-7711-0652-1
PUB DATE	87
NOTE	133p.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC06 Plus Postage.
DESCRIPTORS	*Calculus; Course Descriptions; *Curriculum Development; Foreign Countries; Mathematics Curriculum; *Mathematics Instruction; Secondary Education; *Secondary School Mathematics; Teaching Guides; *Teaching Methods
IDENTIFIERS	*Manitoba (Winnipeg)

ABSTRACT

This manual outlines a calculus course, one of three optional courses designed to meet the needs of students in Manitoba (Canada) secondary schools who demonstrate particular aptitude in mathematics and who desire to study more topics and more advanced topics. Major emphasis is placed on applications and problem solving. The four units in the course concern limits, derivative of a function, applications of derivatives, and integration. For each unit the anticipated number of periods, objectives, notes to teachers, and references to textbooks are included. Appendices list strategies of problem solving and recommendations for school mathematics in the 1980s. (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

FD301438

1987

introduction to calculus 305

optional half credit

interim guide

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

M. L. Yakimishyn

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

manitoba education

SE 050 157

introduction to calculus 305
optional half credit

interim guide

THIS GUIDE IS SUBJECT TO REVISION ON THE BASIS OF INFORMATION RECEIVED FROM SCHOOLS. IT IS EXPECTED THAT MODIFICATIONS WILL NOT BE IN THE DESIGN OF THE PROGRAM. RATHER, CHANGES MAY BE MADE IN AREAS CONCERNING STRATEGIES, RESOURCE MATERIALS, AND IMPLEMENTATION.

manitoba education **1987**

ISBN 0-7711-0652-1

ACKNOWLEDGEMENTS

Sincere appreciation is extended to members of the Development-Writing Committee who planned and wrote the curriculum guidelines for Introduction to Calculus 305.

DEVELOPMENT-WRITING COMMITTEE

Dennis Holmlund (Chairperson) - Pinawa Secondary School, Whiteshell School District No. 2408

Roy Dowling - Department of Mathematics, University of Manitoba

John Farr - Technology Division, Red River Community College

Stephen Kahn - St. Norbert Collegiate, Seine River School Division No. 14

David Klassen - R. D. Parker Collegiate, Mystery Lake School Division No. 2355

Myron Selby - Oak Park High School, Assiniboine South School Division No. 3

Elizabeth Tallin - Dakota Collegiate, St. Vital School Division No. 6

Bob McIntosh, Mathematics Consultant, Curriculum Development and Implementation Branch

Appreciation is also extended to the many mathematics teachers who provided input to the committee and in particular to those who participated in piloting the guide.

Bob Burnell - R.D. Parker Collegiate, Mystery Lake School District No. 2355

Jake Durksen (1984-85)/Barry Booth (1985-86) - Fort Richmond Collegiate, Fort Garry School Division No. 5

Lawrence Gandza (1984-85)/John Tkach (1985-86) - Dauphin Regional Secondary School, Dauphin-Ochre School Division No. 33 (Area No. 1)

Bill Korytowski - Sisler High School, Winnipeg School Division No. 1

Tony Rempel - Steinbach Regional Secondary School, Hanover School Division No. 15

Brian Thorkelson - Vincent Massey High School, Brandon School Division No. 40

Carolyn Wilkinson - Fort Richmond Collegiate, Fort Garry School Division No. 5

Introduction to Calculus 305 is not to be substituted for Mathematics 300.

A student enrolled in Introduction to Calculus 305 should have completed or be currently enrolled in Mathematics 300.

The Introduction to Calculus half-credit course will be of interest to the highly motivated students who wish some introduction to areas of mathematics which will be studied in depth in post-secondary programs. It will be of particular interest to those students planning to register in Engineering, Mathematics, Science, and other mathematics-oriented faculties at out-of-province universities where it is expected that entering students will have some knowledge of the content of calculus. As well, it will facilitate students entry into mathematics-oriented faculties in Manitoba universities and into the two-year technologies in Manitoba community colleges.

CONTENT

OVERVIEW

Rationale 3
Use of Mathematics 3
General Goals of Mathematics 4
General Objectives of Introduction to Calculus 5
Applications Approach 5
Theoretical Approach 5
Problem Solving 5
Why Calculus? 6
Approved Textbooks 7
Supplementary Textbooks 7

INTRODUCTION TO CALCULUS 305

Unit 1: Limits 11
Unit 2: Derivative of a Function 21
Unit 3: Applications of Derivatives 36
Unit 4: Integration 51

APPENDICES

Appendix I: Strategies of Problem Solving 65
Appendix II: Recommendations for School Mathematics in the 1980s 68

overview

OVERVIEW

RATIONALE

Three sequences of mathematics courses are offered in grades 10-12 to meet the needs of the majority of students.

Thd 04 program is designed to concentrate on the application of mathematical skills and concepts to everyday living and is a terminal course.

The 01 program deals with applications to business, trades, and vocational areas and leads to a range of non-mathematics oriented post-secondary training programs.

The 00 program is a theoretical program of algebra, geometry and trigonometry with some consumer-related topics leading to post-secondary training in mathematics and science-oriented courses.

The optional half-credit courses: Statistics and Probability 305, Advanced Mathematics 305, and Introduction to Calculus 305 are designed to meet the needs of students who demonstrate particular aptitude in mathematics and who desire to study more topics and more advanced topics. Increasingly, students are demanding extra preparation in mathematics to enable them to adapt more readily to university courses.

An understanding of elementary probability and statistical theory is advantageous to students entering into business administration, science, psychology, as well as to those planning to specialize in mathematics.

The various topics in Advanced Mathematics and Introduction to Calculus expand the students experience of mathematics by introducing applications of the theory in the 00 programs and also some recent developments in mathematics.

The three optional half-credit courses are of interest to highly motivated students who wish some introduction to areas of mathematics which will be studied in depth in post-secondary programs. These courses are of particular interest to those students planning more advanced studies in Engineering, Mathematics, Science, and other mathematics-oriented faculties at out-of-province universities where it is expected that entering students will have some knowledge of the content of these optional half-credit courses.

The inclusion of these optional half-credit courses will complete the range of departmental programs which will meet the needs of students at all ability and interest levels.

USE OF MATHEMATICS

It is important to study mathematics because of its many applications to everyday life and to specialized scientific work. A knowledge of mathematics is necessary if students are to have an awareness and an understanding of their environment and to enjoy a fuller life.

With this in mind, it is suggested that mathematics be considered:

1. A mode of communications with a language all its own. For students to communicate successfully in mathematics, they must learn the basic concepts without having to meet demands for logical rigour. The language acquired should be used to describe the relationships observed in the real world. At a later stage, symbols should be introduced to record these relationships.

2. A tool. After a mathematical system has been introduced as a language, it should be used as a tool to answer practical questions, thus demonstrating to students that to function in society they need basic skills in arithmetic, algebra, geometry and statistics. Students should be capable of identifying a problem and employing an organized process leading to its solution. This involves applying fundamental operations and information processing skills--such as collecting, organizing and interpreting data--to familiar and unfamiliar contexts. Realistic problems relating to the students' environment should be sought to foster consumer awareness, and vocational and business applications should be stressed at every opportunity.

3. A source of aesthetic enjoyment. Mathematics can be considered an aesthetic discipline like music. Many students can develop an appreciation of mathematics through the discovery of patterns in numbers and in created or naturally occurring designs. Students should be encouraged to reproduce these as well as to create their own. Methodologies and techniques conducive to developing positive inquiring attitudes toward mathematics should be employed.

Students who are most successful in developing skills and understanding concepts in mathematics have learned to recognize and appreciate the three ways in which the subject can be used. Classroom emphasis on these uses cannot always be equal, however, but must vary depending on the nature of the topic and the interests and abilities of the students.

GENERAL GOAL OF MATHEMATICS

The aim of mathematics is to foster maximum and continuous growth in the following behavioral areas: the cognitive (factual), the affective (feelings, perceptions), and the psychomotor (movements). This growth will be accomplished when mathematical content is used to develop processes such as the following:

1. The process of communicating: using mathematical language and symbolism.
2. The process of recalling mathematical skills.
3. The process of comprehending mathematical principles.
4. The process of seeing relationships in mathematics.
5. The process of generalizing: finding common relationships among mathematical concepts.
6. The process of making applications: applying generalizations to new situations.
7. The process of learning self-direction: searching for mathematical ideas.
8. The process of discovering: recognizing mathematical ideas.
9. The process of creating: generating mathematical ideas.
10. The process of appreciating the place of mathematics: understanding its widespread applications to our daily activities.
11. The process of problem solving: identifying, hypothesizing, interpreting, relating, evaluating, concluding.

GENERAL OBJECTIVES OF INTRODUCTION TO CALCULUS 305

The general objectives of the high school mathematics program stated in the previous section apply to Introduction to Calculus 305. Following is a statement of the more specific objectives of the course:

1. To develop an understanding of the properties of functions and their role in many different areas of mathematics.

2. To increase the student's awareness of the scope of mathematics.

3. To prepare the student for further courses in calculus and related fields.

4. To develop skills that will increase a student's ability to simplify algebraic and numeric expressions.

5. To assist students to see that the mathematics of calculus has many practical applications.

APPLICATIONS APPROACH

A central and encompassing approach to making the study of Introduction to Calculus 305 both meaningful and interesting is to place a major emphasis in the area of applications. An application is the process of using a mathematical skill to arrive at a solution to a real life or practical situation. Applications should incorporate interesting, useful, relevant and diverse examples from real life situations. The significance applications is based on relating mathematical concepts and skills to problems encountered in society and the environment. Mathematics may be related to countless aspects of living. Our task is to have students recognize these relations, develop an understanding of the interrelationships that exist within topics of the Introduction to Calculus 305 program and then learn to transfer their use of mathematics to other situations.

Applications, like problem solving, should be integrated into the overall program rather than be dealt with as an independent unit. Whenever possible, integration and coordination with other subject areas should be encouraged. When applications require extensive computations, the use of the calculator or a computer may become a necessary component of the learning process to avoid time-consuming calculations.

THEORETICAL APPROACH

While it is suggested in several sections of this course that formal proof be shown to the students, it should not be an expectation that students will be required to reproduce these proofs. Also, in several sections of this course, proofs are illustrated by intuitive examples rather than by showing formal proofs.

PROBLEM SOLVING

The discipline of mathematics provides one of the best opportunities to practice the process of problem solving. The techniques learned and experience gained are transferable to almost any problem-solving situation and provide the basis of successful living.

Everyone's job involves problem solving. Although many of the problems we have to contend with are repetitious and are easily resolved once the initial solution is known, the most challenging and exciting aspects deal with problems that are completely new to the individual or assist in arriving at solutions in new and creative ways.

A primary purpose of education is to provide a student with the tools to solve a wide variety of problems. Specific problem-solving skills can be taught; however, the methods of successfully tackling untried problems is not always fully understood. There is a sequence of steps by which one can proceed, such as:

1. Fully understanding the problem.
2. Accepting the challenge of searching for a solution.
3. Making a reasonable guess at the answer (this provides a check when a solution is finally found).
4. Pictorially or graphically representing the important aspects of the problem if this is at all possible.
5. Solving a simpler version of the problem or a special case of the more general problem.
6. Using specific cases to discover patterns or different approaches to the problem.
7. Solving the general problem.
8. Checking the solution in the special cases solved before. (Does the solution seem reasonable based on your initial guess?)

The above sequence of steps is helpful in tackling any problem. Considerable intuition and experience are still necessary to tackle the really difficult problems.

The strategies of problem solving are discussed in greater detail in the Appendix.

WHY CALCULUS?

Arnold Toynbee

... at about the age of sixteen, I was offered a choice which, in retrospect, I can see that I was not mature enough, at the time, to make wisely. This choice was between starting on the calculus and, alternatively, giving up mathematics altogether and spending the time saved from it on reading Latin and Greek literature more widely. I chose to give up mathematics, and I have lived to regret this keenly after it has become too late to repair my mistake. The calculus, even a taste of it, would have given me an important and illuminating additional outlook on the Universe, whereas, by the time at which the choice was presented to me, I had already got far enough in Latin and Greek to have been able to go farther with them unaided. So the choice that I made was the wrong one, yet it was natural that I should choose as I did. I was not good at mathematics; I did not like the stuff ... Looking back, I feel sure that I ought not to have offered the choice; the rudiments, at least, of the calculus ought to have been compulsory for me. One ought, after all, to be initiated into the life of the world in which one is going to have to live. I was going to have to live in the Western World ... and the calculus, like the full-rigged sailing ship, is ... one of the characteristic expressions of the modern Western genius.[1]

[1] Experiences, Oxford University Press, pp. 12-13, 1969.

APPROVED TEXTBOOKS

Del Grande, J. and Doff, G. Calculus. Toronto, ON: Gage Publishing, 1979.

Elliott, H.A., et al. Calculus, Complex Numbers and Polar Coordinates. Toronto, ON: Holt, Rinehart and Winston, 1980.

SUPPLEMENTARY TEXTBOOKS

Bittimger, Marvin L. Calculus. Don Mills, ON: Addison-Wesley, 1984.

Edwards, C.H. and Penney, D.E. Calculus and Analytic Geometry. Englewood Cliffs, NJ: Prentice-Hall, 1982.

Loomis, Lynn. Introduction to Calculus. Don Mills, ON: Addison-Wesley, 1985.

introduction to calculus 305

UNIT I: LIMITS (9 - 40 minute periods)

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
The student should be able:			
1. to achieve an understanding of the concept of limits;	Use several examples students might already be familiar with or examples using a graph and a calculator. The concept of limit was first dealt with by the Greek mathematicians Eudoxus and Archimedes who were concerned with finding the area of a circle by circumscribing and inscribing regular polygons, thus trapping the area of the circle between the two polygons. Two thousand years later, a more formal structure was given to the limit by problems arising out of Kepler's work. At first, the notion of indivisibles was suggested by Cavalieri, Wallis and Roberval. Later, Fermat, Newton, Leibniz and Gauchy formalized a more rigorous treatment of limit. Find the limit of the function $2x + 3$ as x approaches 4. A chart like the following may be used.		

X	3.9	3.99	3.999	4.001	4.01	4.1	
2x+3	10.8	10.98	10.998	11.002	11.02	11.2	

It thus appears that the limit of $2x + 3$ as x approaches 4 is 11.

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	ENRICHMENT Asymtotes of an hyperbola. The sum of an infinite geometric progression.		
2. to achieve an understanding of the notation for limits: $\lim_{x \to a} f(x) = b$	The teacher should show why it is reasonable to expect that: 1) $\lim_{x \to 2} (x+1) = 3$ 2) $\lim_{x \to 3} x^2 = 9$ 3) $\lim_{x \to -2} \left(\frac{x^2 - 4}{x + 2}\right) = -4$ Graphs should be part of the explanations. Teachers should explain carefully the use of the equal sign in the limit notation and emphasize that the notation is only a mathematical way of expressing the idea of "approaching."		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
3. to illustrate the following limit theorems:			
a) $\lim_{x \to a} k = k$ where k is a constant	Constant Law: The limit of a constant is that constant.	p. 40	pp. 75-77
	Example:		
	Find the limit of $(7x-12)$ as $x \to 0$.		
	Solution:		
	$\lim_{x \to 0} (7x-12) = \lim_{x \to 0} 7x - \lim_{x \to 0} 12$ $= 0 - 12$ $= -12$		
b) $\lim_{x \to a} x = a$	Identity Law		
c) $\lim_{x \to a} kx = k \lim_{x \to a} x$	Constant times a Function Law		
d) $\lim_{x \to a} x^n = a^n$	Example:		
	Find the limit of x^3 as $x \to 2$.		
	Solution:		
	$\lim_{x \to 2} x^3 = 8$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
e) $\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$	Addition or Subtraction Law: The limit of a sum or difference is the sum or difference of the limits. Example: Find the limit of $(2x^3 - 3x^2 + 2x - 5)$ as $x \to 2$. Solution: $\lim_{x\to 2}(2x^3 - 3x^2 + 2x - 5)$ $= \lim_{x\to 2} 2x^3 - \lim_{x\to 2} 3x^2 + \lim_{x\to 2} 2x - \lim_{x\to 2} 5$ $= 2(2)^3 - 3(2)^2 + 2(2) - 5$ $= 3$	p. 40	pp. 75-77
f) $\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]$	Product Law: The limit of the product is the product of the limits. Example: Find the limit of $(x^2-2x-1)(x^2-2x+4)$ as $x \to 3$. $\lim_{x\to 3}[(x^2-2x-1)(x^2-2x+4)] = [\lim_{x\to 3}(x^2-2x-1)][\lim_{x\to 3}(x^2-2x+4)]$ $= [2][7]$ $= 14$	p. 40	pp. 75-77

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
g) $\lim_{x\to a} [kf(x)] = k[\lim_{x\to a} f(x)]$ (where k is a constant)	The limit of a constant times a function is the constant times the limit of the function. Example: Find the limit of $7x^2$ as $x \to 3$. Solution: $\lim_{x\to 3} 7x^2 = 7[\lim_{x\to 3} x^2]$ $= 7[9]$ $= 63$		
	Note: If p(x) is a polynomial then, $\lim_{x\to a} p(x) = p(a)$. Direct substitution applies for polynomials.	p. 40	pp. 75-79
h) $\lim_{x\to a} [\frac{f(x)}{g(x)}] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$	The limit of a quotient is equal to the quotient of the limit provided the limit of the denominator is not equal to zero. Example: Find the limit of $[\frac{x-3}{x^2-3}]$ as $x \to 2$.	p. 40	pp. 75-79

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Solution: $\lim_{x\to 2}\left[\frac{x-3}{x^2-3}\right] = \frac{\lim_{x\to 2}(x-3)}{\lim_{x\to 2}(x^2-3)}$ $= \frac{-1}{1}$ $= -1$		
	Note: If $p(x)$ and $q(x)$ are polynomials and $q(a) \neq 0$ then $\lim_{x\to a}\left[\frac{p(x)}{q(x)}\right] = \frac{p(a)}{q(a)}$		
	This statement follows from the rule in h) and the note in g).		
4. to determine $\lim_{x\to a} f(x)$, where $f(x)$ is an algebraic function, for certain simple cases;	Students should be given a progressive set of exercises beginning with functions where substitution yields the answer.		
	Example: $\lim_{x\to 2}\left[\frac{3}{x-4}\right] = -\frac{3}{2}$		
	More complicated examples should involve cases where either the denominator approaches zero, or both the numerator and denominator approach zero.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Examples:		
	a) $\lim_{x \to 1} (2x+3)$		
	b) $\lim_{x \to 2} (2x^2+3x-1)$		
	c) $\lim_{x \to 2} \left[\frac{x^3+27}{x+3}\right]$		
	d) $\lim_{x \to -3} \left[\frac{x^3+27}{x+3}\right]$		
	e) $\lim_{x \to 3} \left[\frac{x^2-5x+6}{x-3}\right]$		
	f) $\lim_{x \to 4} \left[\frac{\sqrt{x} - 2}{x - 4}\right]$		
	g) $\lim_{x \to 9} \left[\frac{x - 9}{\sqrt{x} - 3}\right]$		
	h) $\lim_{h \to 0} \left[\frac{1}{h} \left(\frac{1}{2 + 2h} - \frac{1}{2} \right)\right]$		
	i) if $f(x) = x^2 + 3$ evaluate $\lim_{t \to 0} \left[\frac{f(5 + t) - f(5)}{t}\right]$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
	Notes: 1. Students often have difficulty knowing when to write $\lim_{x \to a}$ and when to leave it out. 2. A discussion of the delta, epsilon definition of a limit should be left out.		
5. to demonstrate an understanding of the concept of continuity.	A function $f(x)$ is continuous at a point $x = a$, if: a) $f(a)$ is defined, AND b) $\lim_{x \to a} f(x)$ exists, AND c) $\lim_{x \to a} f(x) = f(a)$ **Notes:** 1. Intuitively, if the graph of a function can be drawn without lifting your pencil from the page, the function will be continuous. 2. Choose many graphic examples before introducing the definition so that students have a good intuitive understanding of the concept.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Examples: 1. Is $x + 2$ continuous at $x = 3$? 2. Is $x + 2$ continuous for all values of x? 3. Is $\frac{5}{x + 2}$ continuous at $x = -2$? 4. Is $\frac{x^2 - 4}{x + 2}$ continuous at $x = -2$? **Remark:** Every polynomial is continuous for all values of x.		
ENRICHMENT: Determine the limit of an algebraic function as the independent variable approaches infinity (increases without bound).	An extension of #4 occurs when the independent variable approaches infinity (increases without bound). **Example:** Show that: $\lim_{x \to \infty} \left[\frac{x^2 + 7}{4x^2 + 1}\right] = \frac{1}{4}$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.

Solution:

$$\lim_{x\to\infty}\left[\frac{x^2+7}{4x^2+1}\right] = \lim_{x\to\infty}\frac{x^2\left(1+\frac{7}{x^2}\right)}{x^2\left(4+\frac{1}{x^2}\right)}$$

$$= \lim_{x\to\infty}\left[\frac{\left(1+\frac{7}{x^2}\right)}{\left(4+\frac{1}{x^2}\right)}\right.$$

$$= \frac{1+0}{4+0}$$

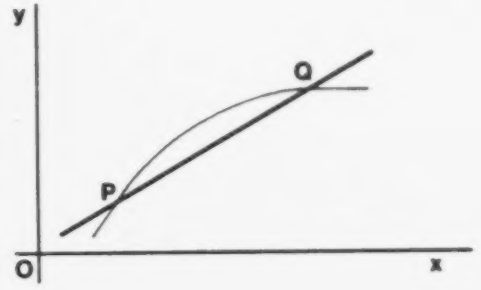
$$= \frac{1}{4}$$

41

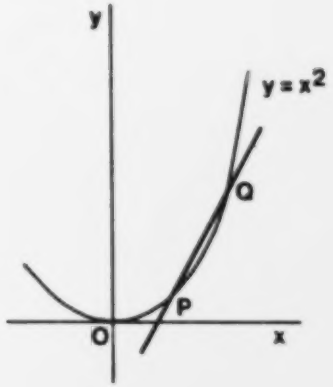
42

UNIT II: DERIVATIVE OF A FUNCTION (27 - 40 minute periods)

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
The student should be able:			
1. to maintain previously developed skills in finding an equation of a line;	Consideration should be given to: 1. oblique lines 2. lines parallel to the axes 3. conditions for parallel and perpendicular lines	pp. 46-48	pp. 1-3
2. to define a secant line for a curve;	A secant line is a line passing through any two points on a curve.	pp. 50-53	p. 3



PQ is a secant line.

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
3. to determine the slope of a secant line;	 The secant line is any line, e.g., PQ that passes through two points on the curve.	pp. 50-53	pp. 4-5 review p. 23, #2

Example:

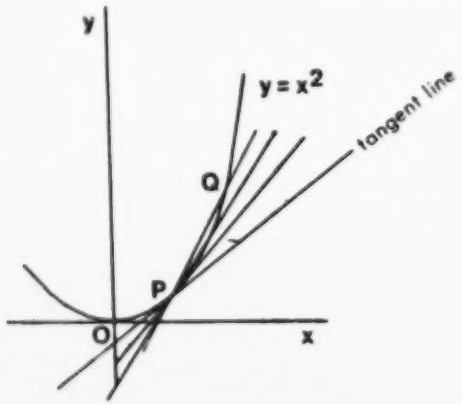
Find the slope of the secant to the parabola $y = x^2$ through the two points where $x = x_1$ and $x = x_2$.

when $x = x_1$ $\quad y = x_1^2$

when $x = x_2$ $\quad y = x_2^2$

The two points are (x_1, x_1^2) and (x_2, x_2^2).

Slope of secant PQ $= \dfrac{x_2^2 - x_1^2}{x_2 - x_1} = x_2 + x_1$.

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
4. to define a tangent line to a curve at a point on the curve;	A tangent to a curve at a point P on the curve is the limit of a sequence of secant lines PQ as Q approaches P along the curve. 	pp. 50-53	pp. 5-7
5. to define the derivative of a function in terms of limits;	The derivative of f at x is $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$	p. 83	pp. 74-75

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
6. to become familiar with a variety of derivative notations;	If $y = f(x)$, then $D_x y = \frac{dy}{dx} = f'(x) = y' = \frac{d}{dx} f(x)$ Note: 1. While there is a variety of notations, the most commonly used are $f'(x)$, y' and $\frac{dy}{dx}$. 2. The slope of the tangent line to a curve $y = f(x)$ at a given point is the value of the derivative of $f(x)$ at that point. Example: Find the slope of the tangent line to the curve $y = x^2$ at the point on the curve where $x = 5$. Solution: For $f(x) = x^2$ $f'(x) = 2x$ The required slope $2(5)$ $= 10$	p. 60 pp. 83-86	pp. 74-75

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
7. to find the derivatives of the various functions using the definition of derivative;	Find the derivatives of the following: Examples: Solutions: a) linear $y = 3x$ $\frac{dy}{dx} = 3$ b) quadratic $y = 4x^2$ $y' = 8x$ c) cubic $y = 2x^3$ $\frac{dy}{dx} = 6x^2$ d) reciprocal $y = \frac{5}{x}$ $y' = \frac{-5}{x^2}$ e) radical $y = \sqrt{x}$ $y' = \frac{1}{2\sqrt{x}}$ f) negative fractional exponent $y = \frac{1}{\sqrt{x}}$ $\frac{dy}{dx} = \frac{-1}{2x^{3/2}}$ Additional examples should include more than one term.	pp. 60-62	pp. 12-19

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
8. to determine the derivatives of algebraic functions by generalizing the results obtained from objective (3);	Example: Given: $y = x^3 + 2x^2 - 3x + 1$, find $\frac{dy}{dx}$ Solution: $\frac{dy}{dx} = 3x^2 + 4x - 3$	pp. 60-64	p. 17, #3 review p. 23, #4 pp. 80-81
9. to apply the rule for finding the derivative of each of the following:	The teacher should derive some of the rules formally but the students are responsible only for applying the rules.		
a) Derivative of a Constant Function			
If $f(x) = k$ (k is a constant) then $f'(x) = 0$.	Example: If $f(x) = 7$, find $f'(x)$. Solution: $f'(x) = 0$	p. 85	p. 79

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
b) <u>Derivativae Rule for an Identity Function</u> If $f(x) = x$, then $f'(x) = 1$.	<u>Example:</u> If $f(x) = x$, find $f'(x)$. <u>Solution:</u> $f'(x) = 1$.		p. 79
c) <u>Power Rule</u> If $f(x) = x^n$ $(n \in I^+)$, then $f'(x) = nx^{n-1}$.	<u>Example:</u> Find the derivative of $f(x) = x^6$ <u>Solution:</u> $f'(x) = (6x^5)$		
d) <u>Derivative Rule for a Constant Times a Function</u> If $f(x) = kg(x)$, then $f'(x) = kg'(x)$ (where k is a constant).	<u>Example:</u> If $f(x) = 5x^3$, find $f'(x)$.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Solution:		
	$f'(x) = 5(3x^2)$ $= 15x^2$		
	The derivative of a constant times a function is the constant times the derivative of the function.		
e) **Derivative Rule for Sums or Differences**	The derivative of a sum or difference is the sum or difference of the derivatives.		
If $F(x) = f(x) \pm g(x)$ then $F'(x) = f'(x) \pm g'(x)$.	**Example:**		
	Find the derivatives of $f(x) = x^4 - 2x^3 + 3x^2 - 4x + 5$		
	Solution:		
	$f'(x) = 4x^3 - 6x^2 + 6x - 4$		
f) **Derivative Rule for Product of Functions**	The derivative of a product of two functions is the first function times the derivative of the second plus the derivative of the first times the second.		
If $F(x) = f(x)g(x)$, then $F'(x) = f(x) g'(x) + f'(x)g(x)$.	**Example:**		
	Find the derivative of $f(x) = (7x^4 + 5x^2 + 8)(2x^4 - 7x^2)$.		
	Solution:		
	$f^1(x) = (7x^4 + 5x^2 + 8)(8x^3 - 14x) + (28x^3 + 10x)(2x^4 - 7x^2)$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Notes: 1. Simplification of derivatives is not required at this time. 2. "Differentiate" and "find the derivatives of" are equivalent statements. 3. Every polynomial $p(x)$ has a derivative for every value of x.		
g) **Derivative Rule for Quotient of Functions** If $h(x) = \frac{f(x)}{g(x)}$, then $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ $\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$ if $g(x) \neq 0$	The derivative of a quotient of functions is the derivative of the numerator times the denominator minus the numerator times the derivative of the denominator and all divided by the square of the denominator. **Example:** Find the derivative of $h(x)$ if $h(x) = \frac{x^2 + 1}{x^3 + 2}$ **Solution:** $\frac{d}{dx}[h(x)] = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] =$ $\frac{(2x)(x^3 + 2) - (x^2 + 1)(3x^2)}{(x^3 + 2)^2}$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
10. to apply the rules introduced in objective (9);	Teachers should have students do sets of exercises which will involve the use of all seven rules introduced in objective #9.	p. 109	p. 96
11. to illustrate the Chain Rule;	The Chain Rule gives us a procedure for finding the derivative of the composition $f[g(x)]$ and is given by: $\frac{d}{dx} f[g(x)] = f'[g(x)]g'(x)$ or If $y = f(u)$ and $u = g(x)$ Then $y = f[g(x)]$ and $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ Example: Find the derivative of $y = (3x^2 + 1)^5$. Solution: $y' = 5(3x^2 + 1)^4(6x)$	pp. 106-107 Select appropriate exercises.	pp. 84-88

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	The derivation of the chain rule could be presented. However, the student should not be expected to derive the rule since the derivation may be too difficult. Students should be expected to perform differentiations using the chain rule in combination with the product or quotient rules. Example: Find the derivative of $y = (3x^2 + 1)^4(2x^2 + 5)$. Solution: $y' = (3x^2 + 1)^4(4x) + 4(3x^2 + 1)^3(6x)(2x^2 + 5)$.		
12. to determine the slope of the tangent line to the graph of the function at a given point;	Use the rules for finding the derivatives. Example: Find the slope of the tangent line to the curve $y = x^2 + 3x$ at the point (1, 4). Solution: slope $m = \frac{dy}{dx} = 2x + 3$	pp. 6-7 50- 55	pp. 8-9

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	when $x = 1$, $m = 2(1) + 3$ $= 5$ $\therefore$ Slope of the tangent line at (1,4) is 5.		
13. to determine the equation of the tangent line to the graph of a function at a given point;	Example: Find the equation of the tangent line to the curve $y = x^2 + 3x$ at the point (1,4). Solution: See example in objective 12. At the point (1,4) the slope of tangent line = m = 5. Equation of tangent line $y - y_1 = m(x - x_1)$ $y - 4 = 5(x - 1)$ $5x - y - 1 = 0$		pp. 20-22
14. to determine derivatives using the technique of implicit differentiation;	Example: Differentiate y with respect to x (or find the derivative of y with respect to x) for the relation $2x^2 - xy - 2y^2 = 1$		pp. 100-105

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Determine the value of the derivative at (-1,1).		
	Does $\frac{du}{dx}$ exist at every point on the curve $2x^2 - xy - 2y^2 = 1$?		
	Solution:		
	$4x - \frac{xdy}{dx} - y - 4y\frac{dy}{dx} = 0$		
	$(x + 4y)\frac{dy}{dx} = 4x - y$		
	$\frac{dy}{dx} = \frac{4x - y}{x + 4y}$		
	At (-1,1), $\frac{dy}{dx} = \frac{4(-1) - 1}{-1 + 4(1)} = \frac{-5}{3}$		
	$\frac{dy}{dx}$ would not exist at any point on the curve $2x^2 - xy - 2y^2 = 1$. That is, at $(\frac{4}{\sqrt{34}}, \frac{-1}{\sqrt{34}}, -\frac{4}{\sqrt{34}}, \frac{1}{\sqrt{34}})$	pp. 112-117	pp. 100-105

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
15. to determine the equation of a tangent line to a curve at a given point when the equation of the curve is given in implicit form;	<u>Example 1</u>: (Same as in objective 14.) Find the equation of the tangent line to the curve $2x^2 - xy - 2y^2 = 1$ at the point $(-1,1)$. <u>Solution</u>: See example in objective 14. Slope of tangent line is: $y - y_1 = m(x - x_1)$ $y - 1 = \frac{5}{3}(x + 1)$ $3(y - 1) = -5(x + 1)$ $3y - 3 = -5x - 5$ $5x + 3y + 2 = 0$ <u>Example 2</u>: Find the equation of the tangent line to the curve $x^2 + y^2 = 25$ at the point $(4, -3)$.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.

Solution:

$$x^2 + y^2 = 25$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At $(4, -3)$, $\frac{dy}{dx} = \frac{-4}{-3} = \frac{4}{3}$

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \frac{4}{3}(x - 4)$$

$$4x - 3y - 25 = 0$$

71

72

Unit III: Applications of Derivatives (21 - 40 minute periods)

While working with the theory of light in the middle of the 18th century, Pierre Maupertuis announced the Principle of Least Action. Although he advocated his principle for theological reasons, modern day mathematicians accept the minimizing or maximizing processes because they are aesthetically appealing and scientifically helpful. It has been shown that nearly all gravitational, optical, dynamical, and electrical laws can be obtained by maximizing or minimizing a function as the time integral of kinetic potential. Albert Einstein showed that maximization of a function, called the interval, accounts for the observed path of the planets. The applications we shall deal with are velocity and acceleration problems, maximum and minimum problems, related rates problems and graphics.

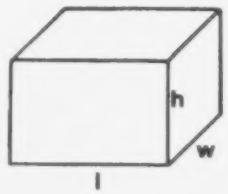
OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
A. MOTION PROBLEMS			
	The concept of acceleration had been known to Nicole Oresme (1325-1382) but did not play a significant role in dynamics until the seventeenth century.		pp. 25-48
The student should be able:			
1. to define average velocity and solve related problems;		pp. 148-151	pp. 36-39
2. to define instantaneous velocity using limits and derivatives;		pp. 148 150	pp. 27 36

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
3. to define average acceleration and solve related problems;		p. 160	pp. 36-39
4. to define instantaneous acceleration using limits and derivatives;		p. 160	pp. 36-39
5. to solve applied motion problems using differentiation.		pp. 160-ffg	pp. 35-36 p. 39 p. 41
B. MAXIMA AND MINIMA			
The student should be able:			
1. to determine the critical values of a function;	A number c called the critical value of the function f(x) if f(c) is defined and either f'(c) = 0 or f'(c) does not exist. Note: Students may require some review in solving polynomial equations to find the zeroes of a polynomial function.	p. 160	pp. 49-52 pp. 40-42

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
2. to identify for a polynomial function the intervals for which the first derivative is positive or negative;	This is the same as stating where the function is increasing or decreasing. A relatively simple way to do this makes use of Armstrong's Rule for signs. The procedure is as follows: Example: State the intervals where the function $f'(x) = -2(x - 3)^2(x + 4)(5 - x)$ is positive. Solution: Step 1: Find $f^1(x)$ in factored form. Step 2: Write the zeroes of $f'(x)$ in order on a number line. $f'(x)$ -4 0 3 5 Step 3: Test a larger value of x in $f'(x)$. Let's say we choose $x = 1000$. Then $f'(x)$ = -2 (positive)(positive)(negative) = positive Step 4: Mark this on the number line + -4 0 3 5		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Step 5: At each zero as you move to the left along the line, alternate the sign if the original factor had an odd exponent or keep the sign if the factor has an even exponent. odd exponents on (x+4), thus change the sign even exponent on (x-3), thus keep the sign odd exponent on (5-x), thus change the sign Step 6: Answer he question. $f'(x)$ is positive for $x < -4$ and $x > 5$		
3. to define the terms, relative maximum," "relative minimum," "absolute maximum," and "absolute minimum."	Students should understand the difference between relative maximum/minimum and absolute maximum/minimum.	p. 160 pp. 195-198 pp. 212-215	pp. 49-52
4. to find for polynomial functions, relative maxima and minima using the first derivative test.	Step 1: Find $f'(x)$ and the critical values of $f(x)$. Step 2: Make a chart using Armstrong's Rule.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
	Step 3: If f'(x) changes from negative to positive as you move to the right then there is a relative minimum at that point: - V + Step 4: If f'(x) changes from positive to negative as you move to the right, then there is a relative maximum at that point: + Λ - Step 5: If f'(x) does not change sign, there is neither a maximum or minimum at that point.		
5. to find the maxima and minima using the second derivative test;	The use of graphs of f(x), f'(x) and f''(x) is an effective teaching strategy for justifying the tests for maximum and minimum. Students should be aware that the second derivative test does not always work.		pp. 113-116
6. to review the formulae for perimeter, area and volume of various plane and solid figures;	Students should not be expected to memorize the formulae. Formulae for circles, quadrilaterals, spheres, cones, rectangular solids, triangles, and cylinders should be mentioned.	pp. 172-177	pp. 53-54 pp. 131-135

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Note: A review of maxima and minima from the standpoint of quadratic functions might prove useful with new solutions developed using differentiation. It is intended that students quickly proceed to questions beyond the scope of quadratic relations.		
7. to solve applied problems;	Example: To send a parcel from Winnipeg to any place in Canada there are restrictions re the size of the parcel. The maximum height cannot exceed 1 metre and the girth (twice the sum of the length and width) cannot exceed 1 metre. Find the dimensions of the box of maximum volume having height of 1 metre and girth of 1 metre. Solution: Step 1: Draw the diagram.  Step 2: Identify variables by marking them on the diagram or by defining them in words. Step 3: Write the formula for the quantity to be maximized or minimized. $V = lwh$ $= lw(1)$ $= lw$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.

Step 4: If the expression for the quantity to be mximized or minimized contains more than one variable, the others must be eliminated by substitution. Look for more information.

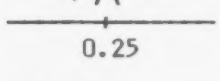
$$2(l+w) = 1 \text{ m}$$
$$\therefore l = 0.5 - w$$

Now substitute and simplify

$$V = lwh$$
$$= (0.5 - w)(w)(1)$$
$$= 0.5\,w - w^2$$

Step 5: Differentiate, solve for critical values and test for maximum/minimum:

$$V = 0.5\,w - w^2$$
$$V' = 0.5 - 2w = 0$$
$$w = 0.25$$



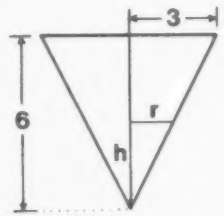
$\therefore$ Max at $w = 0.25$

Step 6: Answer the question. Maximum volume occurs when dimensions of the box are:

length = 0.25 m
width = 0.25 m
height = 1 m

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
C. RELATED RATE PROBLEMS The student should be able:			
1. to use the chain rule to find an expression $\frac{dy}{dx}$ when y is a given function of x and x is a function of t.	This is one of the most difficult concepts for students to understand. Example: If $A = \pi r^2$ find the rate of change of the area with respect to time. Solution: Since there is no variable t in the formula, we must use the chain rule. Ordinary differentiation would yield $A' = 2\pi r$ or $\frac{dA}{dr} = 2\pi r$ If we consider both A and r to be functions of time, however, we have $A(t) = \pi[r(t)]^2$ and $\frac{dA(t)}{dt} = 2\pi[r(t)]\frac{dr}{dt}$ or $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$	pp. 106-107	pp. 124-127

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
2. to interpret a derivative as a rate of change of one variable with respect to another;	Now might be the time to re-emphasize the fact that a derivative is not only the slope of a tangent but also a rate of change. The derivative formula for $\frac{dy}{dx}$ may be written as: $\lim_{\Delta \to 0} \frac{\Delta y}{\Delta x}$ and is read as "the limit of a change in 'y' over a change in 'x' as the change in 'x' approaches 0."	pp. 48,60 66-68	pp. 128-136
3. to solve applied problems using rates.	Express the variable whose rate of change is required in terms of variables for which the rates of change are known.		
	A conical hopper is being filled with wheat at the rate of 0.5 m^3/s. The hopper is 6 m high and 3 m in radius at the top. Find the rate at which wheat is rising in the hopper when the depth is 2 m.	pp. 172 ffg	pp. 124-127
	$\frac{h}{r} = \frac{6}{3}$ $\therefore r = \frac{h}{2}$		



OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Solution: Step 1: Draw the diagram. Step 2: Identify the variables on the diagram or the definitions in words. Step 3: Write down the rates known and those to be found. $\frac{dV}{dt} = 0.5\ m^3/s$ (a rate of change of volume since units are cubic) $\frac{dh}{dt} = ?$ when $h = 2m$ Step 4: Write an equation or formula involving the variables. $V = \frac{1}{3}\pi r^2 h$ Step 5: Count the variables (make sure each is a variable and not a constant) and count the rates. If they do not agree, usually the number of variables has to be reduced by substitution. Look for more information (often similar triangles or Pythagorus Theorem).		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.

$$\frac{h}{r} = \frac{6}{3}$$

Since we want $\frac{dh}{dt}$ we solve for r

$r = \frac{h}{2}$ and substitute this into

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi\left(\frac{h}{4}\right)h$$

$$V = \frac{\pi}{12}h^3$$

Step 6: Differentiate with respect to time.

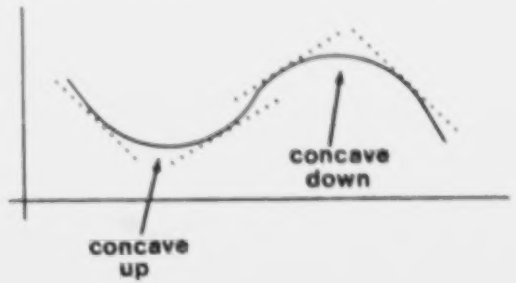
$$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$$

Step 7: Answer the question.

$$0.5 = \frac{\pi}{12}(3)(2)^2\frac{dh}{dt}$$

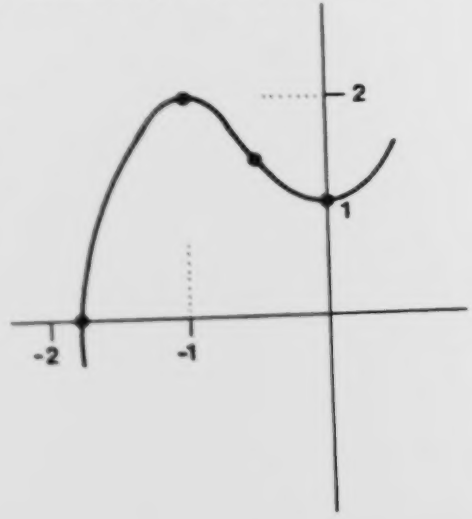
$$\frac{dh}{dt} = \frac{1}{2\pi} \text{ m/s}$$

The wheat is rising at the rate of $\frac{1}{2\pi}$ m/s when the depth is 2m.

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
D. GRAPHING			
The student should be able:			
1. to maintain previous skills of finding x- and y- intercepts, domain, range and relative extrema for a function and where the function is incresing or decreasing;	Choose polynomials that have integral intercepts.	pp. 195-196	pp. 105-113
2. to illustrate graphically the concept of concavity;			
3. to define concave up;	The graph of y = f(x) is said to be concave up for an interval on the x-axis if f'(x) is increasing throughout that interval. Thus, if f''(x) is positive throughout an interval then the graph of y = f(x) is concave up for that interval.		
4. to define concave down;	The graph of y = f(x) is said to be concave down for an interval on the x-axis if f'(x) is decreasing throughout that interval.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
	Thus, if $f''(x)$ is negative throughout an interval then the graph of $y = f(x)$ is concave down for that interval.		
5. to define point of inflection for the graph of $y = f(x)$;	A point, $[c,f(c)]$ on the graph of $y = f(x)$ where a) $f(c)$ is defined b) $f'(c)$ is zero or does not exist c) $f''(x)$ changes sign as x increases through $x = c$ Points of inflection indicate a change of concavity.	p. 219	pp. 113-114
6. to determine concavity and points of inflection;	Example: Find the points of inflection and discuss the concavity of the graph of $f(y) = x^3 - 3x^2 - 24x$. Solution: Step 1: Find $f''(x)$ and solve for its zeroes. $f(x) = x^3 - 3x^2 - 24x$ $f'(x) = 3x^2 - 6x - 24$ $f''(x) = 6x - 6 = 0$ $x = 0$ Step 2: Is $f(1)$ defined? Yes.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Step 3: Test for change of sing at $f''(x)$. Use Armstrong's Rule. $\overset{-}{\underset{}{\quad}}\overset{0}{\underset{}{\quad}}\overset{+}{\underset{}{\quad}}$ Step 4: Answer the question. Point of inflection at $x = 1$ $y = 26$. The curve is concave down at $[x \mid x < 1]$ and concave up when $[x \mid x > 1]$.		
7. to sketch graphs of polynomial functions identifying x- and y-intercepts, domain, range, relative extrema, concavity and any points of inflection.	Example: Graph the polynomial function $y = 2x^3 + 3x^2 + 1$. Identify x- and y- intercepts, domain, range, relative extrema, concavity, and points of inflection. Solution: $y' = 6x^2 + 6x$ $= 6x(x + 1)$ $y'' = 12x + 6$ $= 6(2x + 1)$		



OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
	ENRICHMENT The objective may be extended to include graphs of functions with discontinuities, asymptotes or cusps.		

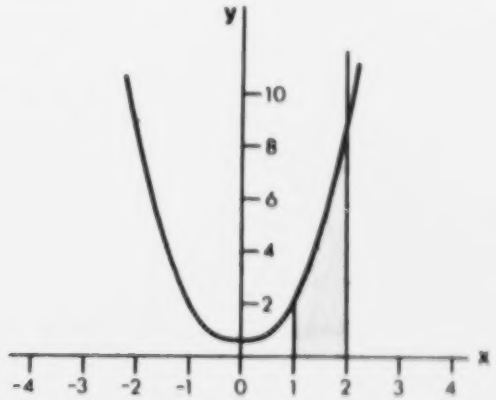
UNIT IV: Integration (18 - 40 minute periods)

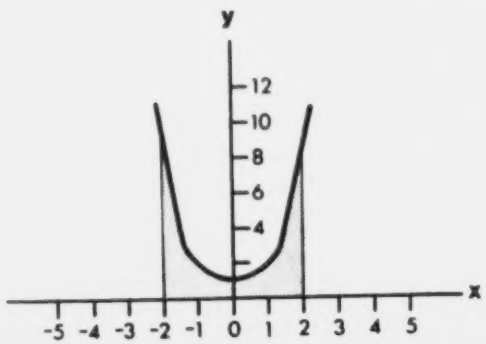
It is interesting to note that the foundations for integral calculus began nearly 2000 years before the 17th century development of differential calculus. The teacher and student may wish to research the developments from the time of Eudoxus (408-355 B.C.) and Archimedes (287-212 B.C.) to the perfection of the integration process by Pierre de Fermat (1601-1655), Gottfried Leibniz (1646-1716), Jakob Bernoulli (1654-1705), and later Augustin Louis Cauchy (1789-1857).

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
The student should be able:			
1. to define indefinite integration;	Indefinite integration is the inverse process of differentiation. The end result of differentiation is the derivative. The end result of integration is the antiderivative or indefinite integral.	p. 307-330	p. 138
2. to find the family of curves whose first derivative has been given;	Limit the questions to polynomial functions and simple rational algebraic functions. Example: Find y if $\frac{dy}{dx} = 4x^3 - 6x + 12$ Solution: $y = x^4 - 3x^2 + 12x + c$	p. 307 ffg	p. 142-145 p. 139-145 p. 165-168
3. to find a specific member of a family of curves given the derivative and the coordinates of a point on a curve;	Example: Find y if $\frac{dy}{dx} = 4x^3 - 6x + 12$ and (2,31) is a point on the graph of y.		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Solution:	pp. 247-250	pp. 142-145
	From objective #2, $y = x^4 - 3x^2 + 12x + c$. Substitute (2,31) for (x,y). Gives c = 3 Then, $y = x^4 - 3x^2 + 12x + 3$		
4. to solve applied motion problems;	Example:		
	An object was thrown vertically upward with an initial velocity (v_0) of 30 m/s. The acceleration of gravity is 9.8 m/s. Calculate (a) the time that would elapse from the initial throwing until the object returned to the ground, (b) the height (h) to which the object would rise.		
	Solution:		
	Upward velocity after t seconds:		
	$v = v_0 - gt$		
	or $\frac{dh}{dt} = v_0 - gt$		
	Integrating, we get $h = v_0 t - \frac{1}{2}gt^2 + c$		
	When t = 0, h = 0		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	$\therefore 0 = 0 - 0 + C$ or $C = 0$ $\therefore h = v_0 t - \frac{1}{2}gt^2$ a) When object reaches ground $h = 0$, substituting gives $0 = v_0 t - \frac{1}{2}gt^2$ or $(v_0 - \frac{1}{2}gt)t = 0$ $\therefore t = 0$ or $t = \frac{2v_0}{g}$ $t \doteq \frac{2(30)}{9.8}$ $t \doteq 6.15$ b) Object would rise for approximately 3 seconds $h = v_0 t - \frac{1}{2} gt^2$ $= 30(3) - \frac{1}{2}(9.8)(3)^2$ $h = 45.9$ m		1.8

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
5. to define and evaluate definite integrals;	<u>Example:</u> Evaluate $\int_2^5 x^2 dx$ <u>Solution:</u> $\int_2^5 x^2 dx = \frac{x^3}{3}\Big|_2^5 = \frac{125}{3} - \frac{8}{3} = \frac{117}{3}$	p. 332	pp. 159-165
6. to determine: a) for the case where $f(x) \geq 0$, determine the area between the curve $y = f(x)$ and the x-axis, between the lines $x = a$ and $x = b$ where $(a < b)$;	<u>Examples:</u> 1. Find the area between $y = 2x^2 + 1$ and the x-axis between $x = 1$ and $x = 2$.  	p. 336	pp. 150-156

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Solution: $\int_1^2 (2x^2 + 1)dx = [\frac{2}{3}(x)^3 + x]_1^2$ $= [\frac{16}{3} + 2] - [\frac{2}{3} + 1]$ $= \frac{22}{3} - \frac{5}{3}$ $= \frac{17}{3}$		
b) for the case in which x changes sign between x=a and x=b, determine the area between the curve y=f(x) and the x-axis and between the lines x=a and x=b;	**Example:** Find the area between $y = 2x^2 + 1$ and the x-axis and between the lines $x = -2$ and $x = 2$. 	p. 336	pp. 150-156

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.

Solution:

$$\int_{-2}^{2} (2x^2 + 1)dx = \left[\frac{2}{3}x^3 + x\right]_{-2}^{2}$$

$$= \left[\frac{2}{3}(2^3) + 2\right] - \left[\frac{2}{3}(-2^3) + -2\right]$$

$$= \frac{22}{3} + \frac{22}{3}$$

$$= \frac{44}{3}$$

Note: Teachers may wish to demonstrate that if $y = f(x)$ lies directly below the x-axis then

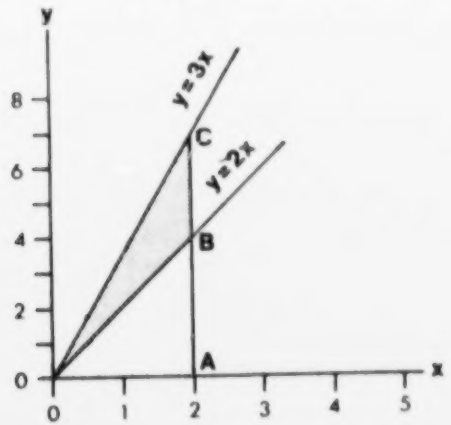
$$\text{Area} = -\int_a^b f(x)dx.$$

Suppose $a < b$.

7. to determine the area between two curves;

For the case where $f(x) > g(x)$ between a and b, the area (A) of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$ and the lines $x = a$ and $x = b$ is:

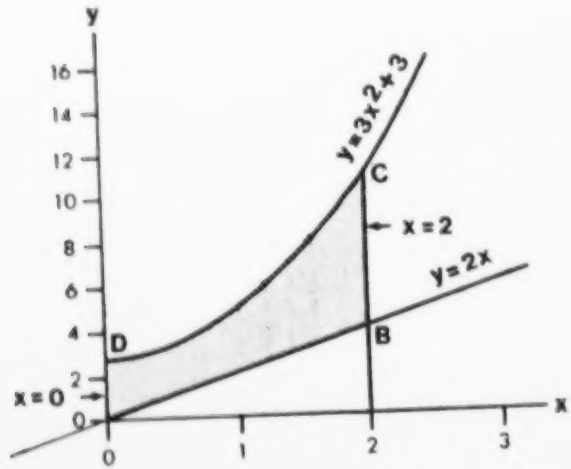
$$A = \int_a^b [f(x) - g(x)]\, dx$$

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
a) between two lines over a given interval;	Example: Find the area between $y = 2x$ and $y = 3x$ from $x = 0$ to $x = 2$. Solution: 1. Sketch the curves and identify the region.  2. Substitute $A = \int_0^2 [f(x) - g(x)]\, dx$ $A(\Delta OBC) = \int_0^2 (3x)dx - \int_0^2 (2x)dx$		116

115

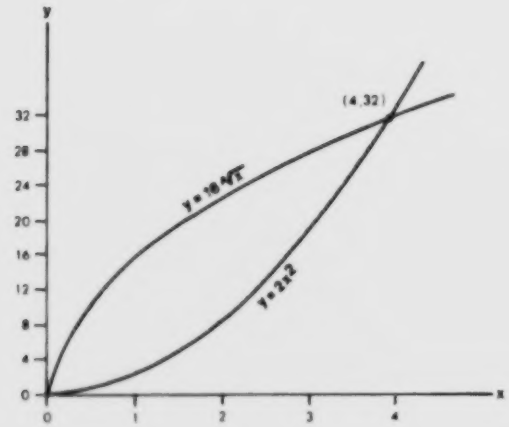
OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	$= \left[\frac{3x^2}{2}\right]_0^2 - \left[x^2\right]_0^2$ $= 6 - 4$ $= 2$ or Area ΔOBC = Area ΔOAC - Area ΔOAB Area $(\Delta OAC) = \int_0^2 (3x)dx$ $= \left[\frac{3x}{2}\right]_0^2$ $= 6 - 0$ $= 6$ Area $(\Delta OBC) = \int_0^2 (2x)dx$ $= \left[x^2\right]_0^2$ $= 4$		

OBJECTIVES	NOTES TO TEACHERS	TEXT REF.	
		Del Grande et al.	Elliott et al.
	Area (ΔOBC) = 6 - 4 = 2		
b) between a line and a curve over a given interval;	Example: Find the area bounded by the graphs of $y = 3x^2 + 3$ and $y = 2x$ and the lines $x = 0$ and $x = 2$.	pp. 302, 336-337	pp. 169-175



Required: Area of OBCD

OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	Solution:		
	Area (OBCD) $= \int_0^2 (3x^2 + 3)dx - \int_0^2 (2x)dx$		
	$= [x^3 + 3x]_0^2 - [x^2]_0^2$		
	$= [(8 + 6) - (0 + 0) - [4 - 0]$		
	$= 14 - 4$		
	$= 10$		
c) enclosed between two curves;	**Example:**		
	Find the area between the graphs of $y = 2x^2$ and $y = 16\sqrt{x}$.	p. 302 pp. 336-337	pp. 169-175



OBJECTIVES	NOTES TO TEACHERS	TEXT REF. Del Grande et al.	TEXT REF. Elliott et al.
	<u>Solution:</u> Find the point of intersection of the curves $y = 16\sqrt{x}$ and $y = 2x^2$. Solving: $x = 0$, $y = 0$ and $x = 4$, $y = 32$ $A = \int_0^4 (16\sqrt{x})dx - \int_0^4 (2x^2)dx$ $= [\frac{2}{3}(16)x^{\frac{3}{2}}]_0^4 - [\frac{2}{3}x^3]_0^4$ $= [\frac{256}{3}] - [\frac{128}{3}]$ $= \frac{128}{3}$		

appendices

APPENDIX I

STRATEGIES OF PROBLEM SOLVING

In the teaching/learning of problem-solving, an instructional approach should be used which helps students learn and choose procedures for solving problems. These procedures are easy to state and recognize, but they are often quite elusive when teaching. Difficulty frequently exists when teaching problem solving because, unlike the teaching of computational skills or concepts, there is no specific content involved. In problem solving, an individually acquired set of processes is brought to bear on a situation that confronts the individual.

There are generally four procedures (stages) which appear inherent in problem-solving. These procedures, their descriptions and associated strategies have been compiled and adapted from a variety of sources and authors (George Polya, J.F. LeBlanc, Ohio Department of Education, Math Resource Project, 1980, NCTM Yearbook) and are listed below:

1. Understand the Problem

   What is the problem? What are you trying to find? What is happening? What are you asked to do?

   Suggested Strategies:

   - Paraphrase the problem or question (Restate the problem in your own words to internalize what the problem entails.)
   - Identify wanted, given and needed information (Help students focus on what is yet to be determined from the problem statement as well as list information so that they may better be able to discover a relationship between what is known and what is required.)
   - Make a drawing (May help to depict the information of a problem, especially situations involving geometric ideas.)
   - Simulate situation (Helps to picture how the problem actions occur and how they are related thereby giving a better understanding of the problem.)
   - Check for hidden assumptions (What precisely does the problem say or not say? Are you assuming that may not be implied? Beware of mistaken inferences.)

2. Devise a Plan to Solve the Problem

   What operations should you use? What do you need to do to solve the problem? How can you obtain more information or data to seek the solution?

   Suggested Strategies:

   - Solve a simpler (or similar) problem (Momentarily set aside the original problem to work on a simpler or similar case. Hopefully the relationship of the simpler problem will point to the solution for the original problem.)
   - Construct a table (Organizing data in tabular form makes it easier to establish patterns and to identify information which is missing.)

- Look for a pattern or trend (Does a pattern continue or exist? In connection with the use of a table, graph, etc., patterns or trends may be more apparent.)

- Solve part of the problem (Sometimes a series of actions each dependent upon the preceding one, is required to reach a solution. Similarly, it may be that certain initial actions will either produce a solution or uncover additional information to simplify the task of solving the problem.)

- Make a graph or numberline (May help organize information in such a way that it makes the relationship between given information and desired solution more apparent.)

- Make a diagram or model (When using the model strategy attempt to select objects or actions to model those from the actual problem that represents the situation accurately and enables you to relate the simplified problem to the actual problem. May be used in connection with, or in place of other similar strategies, i.e., acting out the problem.)

- Guess and check (Guessing for a solution should not be associated with aimless casting about for an answer. The key element to this strategy is the "and check" when the problem solver checks his guesses against the problem conditions to determine how to improve his guess. This process is repeated until the answer appears reasonable. An advantage of this "guess and check" strategy is that it gets the individual involved in finding a solution by establishing a starting point from which he/she can progress. Used constructively with a table or graph this strategy may be a valuable tool.)

- Work backwards (Frequently, problems are posed in which the final conditions of an action are given and a condition is asked for which occurred earlier or which caused the final outcome. Under these circumstances working backwards may be valuable.)

- Change your point of view (Some problems require a different point of view to be taken. Often one tends to have a "mind set" or certain perspective of the problem which creates a difficulty in discovering a solution. Frequently, if the first plan adopted is not successful, the tendency is to return to the same point of view and adopt a new plan. This may be productive, but might also result in continuous failure to obtain a solution. Attempt to discard previous notions of the problem and try to redefine the problem in a completely different way.)

- Write an equation (Often in conjunction with other strategies--using a table, diagram, etc.--one selects appropriate notation and attempts to represent a relationship between given and sought information in an open sentence.)

3. Carry Out the Plan

For some students the strategy/strategies selected may not lend itself/themselves to a solution. If the plan does not work, the problem-solver should revise the plan, review Stage 1, and/or try another plan or combination of plans from Stage 2.

4. Look Back At What Has Been Done (Consolidating Gains)

Is the result reasonable and correct? Is there another method or solution? Is there another solution? Is obtaining the answer the end of the problem?

- Generalize (Obtaining an answer is not necessarily the end of a problem. Re-examination of the problem, the result and the way it was obtained will frequently generate insights far more significant than the answer to the specific situation. It may enable the student to solve whole classes of similar and even more difficult problems.)

- Check the solution (The very length of a problem or the fact that symbolic notation is used may tend to make one lose sight of the original problem. Does the answer appear reasonable, does it satisfy all the problem requirements.)

- Find another way to solve it (Can you find a better way to confront and deal with the problem? The goal of problem-solving is to study the processes that lead to solutions to problems. Once a solution is discovered, search the problem for further insights and unsuspected ideas and relationships.

- Find another solution (Students tend to approach many problem situations with the expectation of only one correct solution. In many practical, daily life situations there may be many answers that are correct and acceptable.)

- Study the solution process (Studying the process of solution makes the activity of problem solving more than answer-getting and can expand an individual problem into a meaningful total view of a family of related problems.)

- Communicate your solution to others (Solutions should be stated in a clear and precise manner. Solutions should include units and other pertinent information from the problem.)

It may be noted that the four stages of the above model are not necessarily discrete. For example, one may move without notice into Stage 2 while attempting to generate more information to understand the problem better.

If the four-stage model is used, the key is to select an appropriate strategy or strategies to help answer the questions suggested by each stage. The strategies listed, and those devised by students will hopefully alter the problem information, organize it, expand it, and make it more easily understood. Strategies then may be thought of as the tools of problem solving and the four-stage model, the blueprint.

APPENDIX II

RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980s

The National Council of Teachers of Mathematics recommends that:

1. problem solving be the focus of school mathematics in the 1980s;

2. basic computer skills in mathematics be defined to encompass more than computational facility;

3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;

4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;

5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;

6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;

7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;

8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.